Answer any 5 questions **5x5 = 25**

1. Solve the partial differential equation using Lagrange's method: $p \tan x + q \tan y = \tan z$ where $z = z(x, y), p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$. **5**

2. Solve the partial differential equation using Lagrange's method of multipliers: $x^2(y-z)p + y^2(z-x)q = z^2(x-y)$where $z = z(x, y), p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$. **5**

3. a. Form the partial differential equation by eliminating the arbitrary constants from the equation: $z = xy + y\sqrt{x^2 - a^2 + b^2}$ where $z = z(x, y)$. **2**

3. b. Form the partial differential equation by eliminating the arbitrary function from the equation: $z = x^n f\left(\frac{y}{x}\right)$ where $z = z(x, y)$. **3**

4. Solve the partial differential equation: $y^2 p - xyq = x(z - 2y)$ where $z = z(x, y), p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$. **5**

5. Suppose two objects of mass m_1 and m_2 where $m_1 > m_2$ are falling from rest, from the same initial position, through a resistive medium under constant gravitational force. Assume that the resistance is proportional to the object's speed, that no other forces are in action and the motion is considered in one dimension only. Formulate the differential equation of this system. Show that the heavier object will fall faster than the lighter object. **5**

6. Solve the initial value problem using the method of Euler-Cauchy: $x^2 y'' - 4xy' + 6y = 0, y(2) = 0, y'(2) = 4$. **5**

7. Find a linearly independent solution by the method of reduction of order for the differential equation: $(x^2 + 1)y'' - 2xy' + 2y = 0$, given $y = x$ is one solution. **5**

8. Solve the homogeneous differential equation: $xy\frac{dy}{dx} - y^2 = (x + y)^2 e^{-\frac{y}{x}}$. **5**