

Choose the single correct answer for the questions below. Answer all questions and show the solution process. (5 × 2 = 10)

1. Which option is the correct general solution of the differential equation below?

$$\frac{2x}{y^3} dx + \frac{y^2 - 3x^2}{y^4} dy = 0.$$

(A) $\frac{x^2}{y^3} + \frac{1}{y} = C$

(B) $\frac{x^2}{y^3} - \frac{1}{y} = C$

(C) $\frac{x^2}{y^2} + \frac{1}{y} = C$

(D) $\frac{x^2}{y^3} + y = C$

2. Which option is the correct general solution of the differential equation below?

$$y^2 + \left(x - \frac{1}{y}\right) \frac{dy}{dx} = 0.$$

(A) $x - 1 = \frac{1}{y} + Ce^{1/y}$

(B) $x - 1 = \frac{1}{y} + Ce^{-1/y}$

(C) $x - 1 = \frac{1}{y} + Cy$

(D) $x - 1 = \frac{1}{y} + Ce^y$

3. Let $y_1(x)$ and $y_2(x)$ be two linearly dependent solutions of the differential equation

$$y'' + p(x)y' + q(x)y = 0.$$

Here $p(x), q(x)$ are continuous on an interval I and $y = y(x)$. If W refers to Wronskian, which option below is true?

(A) $W(y_1, y_2)(x) \neq 0$ for all $x \in I$

(B) No conclusion can be made

(C) $W(y_1, y_2)(x)$ may or may not be zero

(D) $W(y_1, y_2)(x) = 0$ for all $x \in I$

4. Which option is the correct linear first-order partial differential equation obtained by eliminating the arbitrary function from the equation below?

$$z = x + y + f(xy),$$

Here $z = z(x, y)$, $p = \frac{\partial z}{\partial x}$, and $q = \frac{\partial z}{\partial y}$.

(A) $xp + yq = x + y + 2xy$

(B) $xp + yq = x + y$

(C) $xp - yq = x - y$

(D) $xp - yq = x + y$

5. Which option is the correct general solution of the partial differential equation below?

$$(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$$

Here $z = z(x, y)$, $p = \frac{\partial z}{\partial x}$, and $q = \frac{\partial z}{\partial y}$.

(A) $F(x + y + z, x^2 + y^2 + z^2) = 0$

(B) $F(xy + yz + zx, xyz) = 0$

(C) $F(x + y, y + z) = 0$

(D) $F\left(\frac{x - y}{y - z}, \frac{y - z}{z - x}\right) = 0$