

1. Answer any FIVE questions out of the followings $5 \times 2 = 10$

- a) If $I_n = \int \log^n x \, dx$ where $n \geq 1$, find the expression for I_4 using the reduction formula for I_n . 2
- b) Suppose $I_n = \int \tan^n x \, dx$ where $n \geq 2$. Find the value of $\int_0^{\pi/4} \tan^4 x \, dx$ using the reduction formula for I_n . 2
- c) Let $I_n = \int \csc^n x \, dx$ where $n \geq 2$. Find the expression for I_3 using the reduction formula for I_n . 2
- d) Show that a real square matrix can be uniquely expressed as the sum of a symmetric and a skew symmetric matrix. 2
- e) If A be a real skew symmetric matrix of order n and P be a real $n \times 1$ matrix, prove that $P^t A P = 0$. 2
- f) Without expanding the determinant, prove that $\begin{vmatrix} 0 & -a & -b \\ a & 0 & c \\ b & -c & 0 \end{vmatrix} = 0$. 2
- g) Find the asymptotes of the curve $x^3 - 6x^2y + 11xy^2 - 6y^3 + x + y + 1 = 0$. 2
- h) Let $y = \sin^3 x$. Find y_n . 2

2. Answer any THREE questions out of the followings $3 \times 5 = 15$

- a) i. Solve the integral $\int_0^1 \frac{x^4}{\sqrt{1-x^2}} \, dx$ using a suitable reduction formula. 3
 ii. Find the value of $\int_0^{\pi/2} \cos^4 x \, dx$. 2
- b) i. Solve $\int_0^1 \frac{x^5}{1+x^2} \, dx$ using a suitable reduction formula. 3
 ii. Solve $\int_{\pi/4}^{\pi/2} \cot^2 x \, dx$ using a suitable reduction formula. 2
- c) Expand by Laplace's method to prove that $\begin{vmatrix} a & b & c & d \\ -b & a & d & -c \\ -c & -d & a & b \\ -d & c & -b & a \end{vmatrix} = (a^2 + b^2 + c^2 + d^2)^2$. 5
- d) If $y = e^{a \sin^{-1} x}$, prove that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 + a^2)y_n = 0$. Also find the value of $y_n(0)$. 5